Sparse matrices

Large Scale Systems Course

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Questions on Sparse matrices

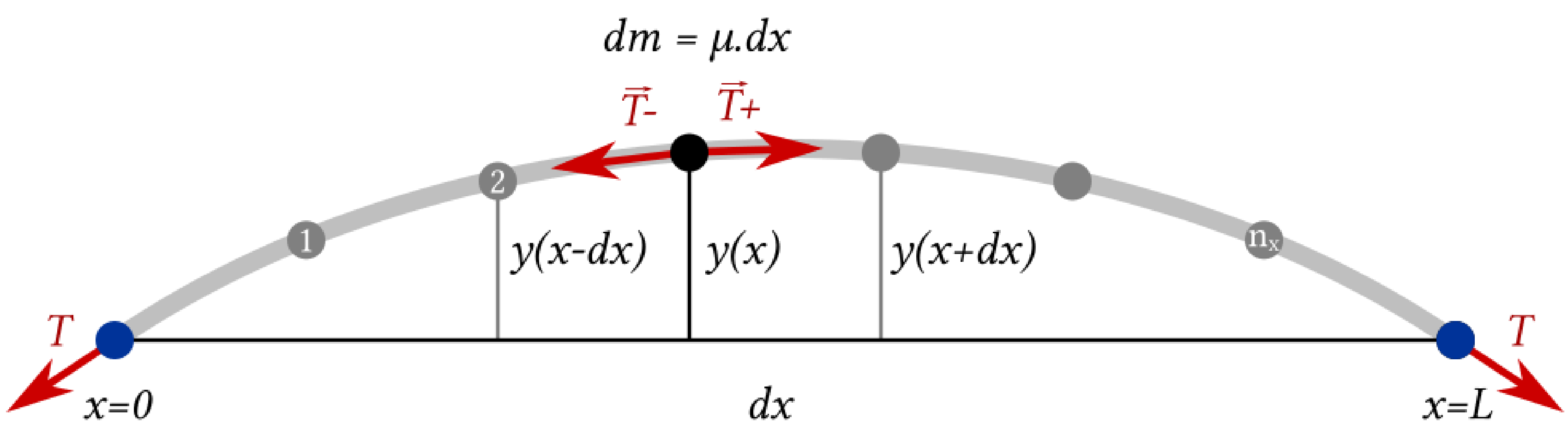
- How to store sparse matrices efficiently?
- What speed gain on computations ?

Physical example

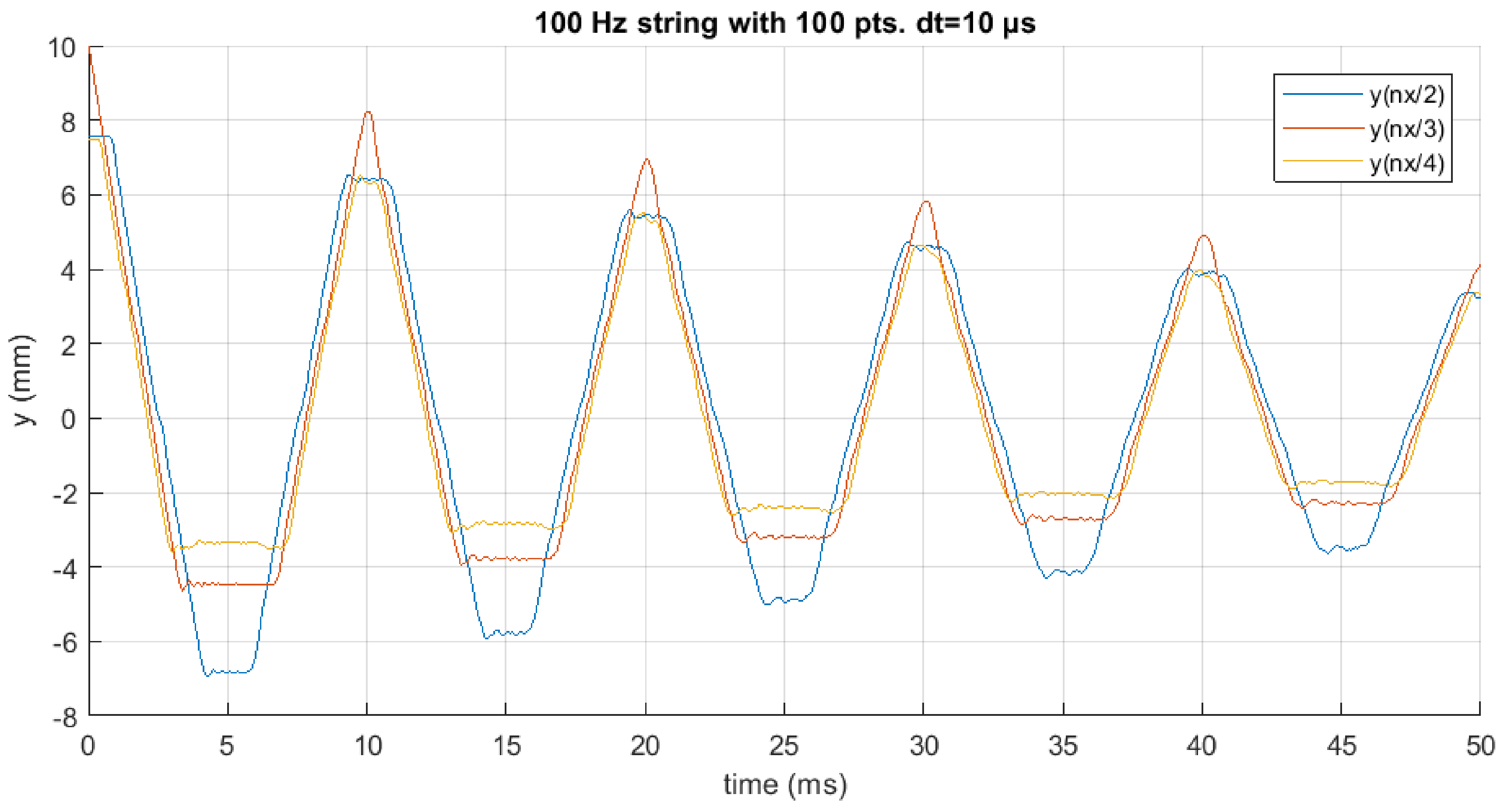
- One application where sparse matrices occurs frequently (always?): simulations of physical systems with **Finite Difference/Elements**.
- We consider here a 1D system : a guitar's string
  - simple enough to be simulated with dense matrices
  - to make the comparison with sparse matrices

String modeling

- Continuous string, modeled with wave equation (Partial Derivative Eq.) has solution y(x,t)
- String is **discretized** in n_x points $\longrightarrow$ large ODE.



Time domain simulation

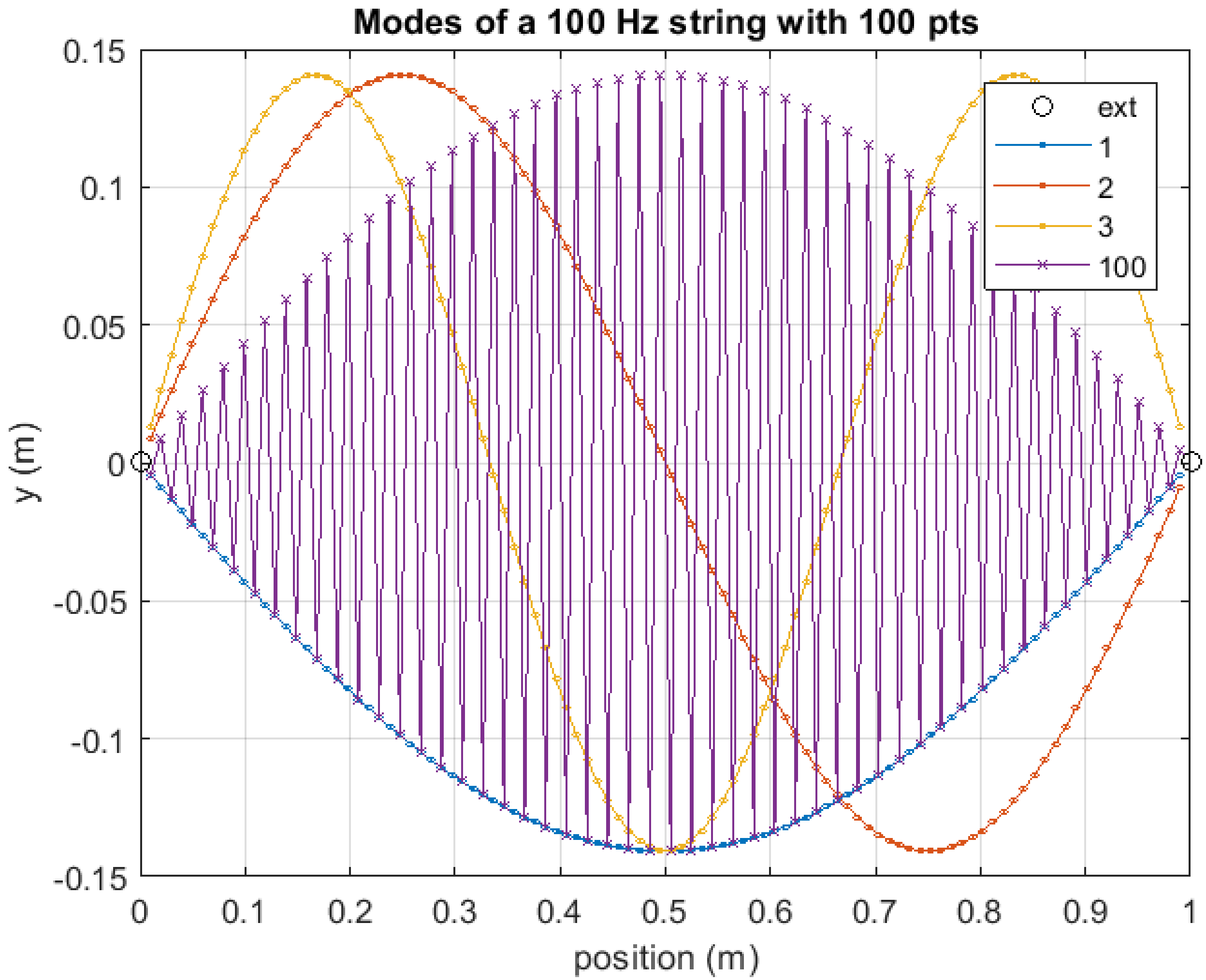


See also animation of the entire string along time

Modal analysis

Modes:

- f_1 = 100 Hz
- f_2 = 200 Hz
- …
- f_{nx}, with n_x the nb of discrete points



Linear algebra requirements

Physical analysis	**Linear algebra operation**
Time domain simulation with an explicit* ODE integration	Matrix-vector product *compute b := A.x*
Frequency response (Bode diagram)	Solve linear matrix equation *find x such that A.x = b*
Modal analysis	Compute eigenvalues and eigenvectors (or a small subset of those) *find (x, λ) such that A.x = λ.x*

Linear algebra operations needed for the numerical analysis of the vibrating string

* *Implicit* ODE integration requires solving linear matrix equations

Assignment

- For each of the 3 linear algebra operation:
  - Compare computation times between dense and sparse versions, for a range of n_x values: 100, 200, 500, 1000… (as long as time keeps reasonable)
  - Does those time increase asymptotically with $(n_x)^3$ or a similar power law?
  - Where is the break-even: for which value of n_x is it more interesting to use sparse computation?
- For a 1 second simulation budget, what is the biggest value of n_x that can be used for:
  - a time simulation with $n_t = 10^3$ pts?
  - a Bode diagram with $n_f = 10^3$ pts?
  - a modal analysis with 10 modes?

You can start with the `timing.m` Matlab script